

NATIONAL INSTITUTE OF TECHNOLOGY DURGAPUR

Odd Semester Mid-term Examination, 2023-24

Course Code: MAC01

Full Marks: 25

Course Name: Mathematics-I

Time: 1.5 Hrs

Answer all questions. Symbols have their usual meanings.

1. Use Mean Value Theorem to show that $\sqrt[3]{28}$ lies between $3 + \frac{1}{28}$ and $3 + \frac{1}{27}$. 5 [CO#1]
2. By Taylor's theorem prove that $x - \frac{x^3}{3!} < \sin x < x - \frac{x^3}{3!} + \frac{x^5}{5!}$ for $x > 0$. 5 [CO#1]
3. Test the absolute and conditional convergence of the series 5 [CO#4]

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{\sqrt{n+1}}, \quad x \in \mathbb{R}.$$

P.T.O.

4. Let

$$f(x, y) = \begin{cases} x \sin \frac{1}{y} + y \sin \frac{1}{x}, & xy \neq 0 \\ 0, & xy = 0. \end{cases}$$

Find the limit at the origin. Also find the repeated limits at origin.

5 [CO#1]

Or

Obtain the Taylor's expansion of

$$f(x, y) = x^2y + 2y + 3$$

in powers of $x + 1$ and $y - 1$.

5 [CO#1]

5. If $f(x, y) = \tan^{-1} \frac{x^3 + y^3}{x - y}$, using Euler's theorem, show that

5 [CO#1]

$$x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy} = (2 \cos 2f - 1) \sin 2f.$$

NATIONAL INSTITUTE OF TECHNOLOGY DURGAPUR
Odd Semester Mid-Term Examination, 2023-24

Course Code: MAC331

Full Marks: 25

Course Name: Mathematics-III

Time: 1.5 Hours

Question Paper No.: NITDGP/MAC331/

Date of Exam: 15-09-2023

Symbols have their usual meanings. Brief and to the point answers are preferred.

Instruction: Answer **ALL** questions.

Question No.	Body of the Question	Marks	Mapped CO
1	Form partial differential equations by eliminating the arbitrary functions F, f , and g from the relations (a) $F(xy + z^2, x + y + z) = 0$, and (b) $z = f(x + 3y) + g(x - 3y)$.	3+2	CO 1
2	Find the general solution of the PDE $(D - 2D')^2(D - D')^2z = e^{2x+y}$.	5	CO 1
3	Solve: $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$.	5	CO 1
4	Find the bilinear transformation which maps the points $(0, i, \infty)$ onto $(1, 3, -i)$. or Determine an analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$.	5 5	CO 3

Question No.	Body of the Question	Marks	Mapped CO
5	<p>Consider the functions f and g defined by</p> $f(z) = \sqrt{ xy }$ <p>and</p> $g(z) = \begin{cases} \frac{xy}{x^2+y^2}, & z \neq 0, \\ 0, & z = 0. \end{cases}$ <p>Check whether: (a) f and g satisfy Cauchy-Riemann equations at $z = 0$, and (b) $f'(0)$ and $g'(0)$ exist?</p>	5	CO 3

Course Outcomes:

CO1: Acquire the idea about mathematical formulations of phenomena in physics and engineering.

CO2: To understand the common numerical methods to obtain the approximate solutions for the intractable mathematical problems.

CO3: To understand the basics of complex analysis and its role in modern mathematics and applied contexts.

CO4: To understand the optimization methods and algorithms developed for solving various types of optimization problems.